Math 371 **Name:** ______________________
Spring 2020
Practice 1
2/20/2020
Time Limit: 80 Minutes ID ______________________

"My signature below certifies that I have complied with the University of Pennsylvania's Code of Academic Integrity in completing this"

Signature ______________________

This exam contains 9 pages (including this cover page) and 6 questions.
Total of points is 70.

- Check your exam to make sure all 9 pages are present.
- You may use writing implements on both sides of a sheet of 8"x11" paper.
- NO CALCULATORS.
- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Good luck!

Grade Table (for teacher use only)

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	20	
Total:	70	

1. (10 points) Define a symmetric bilinear form on $\mathbb{R}^3$ by $\langle X, Y\rangle = X^T A Y$ where $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find a basis v_1, v_2, v_3 such that $\langle v_i, v_j\rangle = 0$ for all $i \neq j$.

2. (10 points) Find an injective group homomorphism from $U(1)$ to $SU(2)$.

3. (10 points) Let $A \in U(n)$ be a unitary matrix. Let v_1 and v_2 be two eigenvectors with distinct eigenvalues λ_1 and λ_2. Prove that $\langle v_1, v_2 \rangle = v_1^* v_2 = 0$

4. (10 points) Prove that two elements A, B in unitary group $U(2)$ are in the same conjugacy class if and only if $trace(A) = trace(B)$ and $det(A) = det(B)$.

5. (10 points) Construct a one dimensional group representation $R\colon C_n \to GL(1)$ of cyclic group C_n of order n such that $ker(R) = e$.

6. (20 points) Let V be the vector space of traceless 2×2 real matrices $\{A \in M_{2\times 2}(\mathbb{R}) | trace(A) = 0\}$.

   (a) Prove that $\langle A, B \rangle = trace(A^T B)$ defines a positive definite symmetric bilinear form on V.

   (b) Prove that $P \cdot A = PAP^T$ defines a linear operation of $SO(2)$ on V.

   (c) Use the previous two parts to define a group homomorphism from $SO(2)$ to $SO(3)$.

   (d) Find the kernel of this homomorphism.

Draft 1:

If you use this page and want it looked at, then you must indicate so on the page with the original problem on it. Make sure you label your work with the corresponding problem number.

Draft 2:

If you use this page and want it looked at, then you must indicate so on the page with the original problem on it. Make sure you label your work with the corresponding problem number.